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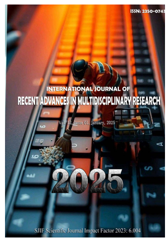

RESEARCH ARTICLE

NEUTROSOPHIC EXPANSION AND FIXED POINT RESULT FOR NEUTROSOPHIC EXPANSION ON NEUTROSOPHIC METRIC SPACE (NMS)

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ABSTRACT

Kirişci M., Simsek N., Akyigit M.(21) have established fixed point result for neutrosophic Banach contraction. The aim of this paper is to put the notion of Neutrosophic Expansion on Neutrosophic Metric Space, and to prove a fixed point theorem for a Neutrosophic Expansion mapping.

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INTRODUCTION

The concept of Fuzzy Sets introduced by Zadeh (1) has attracted all the scientific fields since its starting. It is seen that this concept remained failed for real-life situations, to provide enough solution to some problems in time.Atanassov (2) put the idea of Intuitionistic fuzzy sets for such cases. Neutrosophic set (NS) is a new version of the idea of the classical set which is defined by Smarandache (3). Some of other generalizations are FS (1) interval-valued FS (4), IFS (2), interval-valued IFS (5), the sets paraconsistent, dialetheist, paradoxist, and tautological (6), Pythagorean fuzzy sets (7). Combining the concepts Probabilistic metric space and fuzziness, fuzzy metric space (FMS) is introduced in (8). Kaleva and Seikkala (9) have defined the fuzz metric as the nearness between two points with respect to a real number to be a non-negative fuzzy number. In (10) some basic properties of FMS studied and the Baire Category Theorem for FMS proved. Further, some properties of metric structure like separability, countabilityetc are given and Uniform Limit Theorem is proved in (11). Afterward, FMS has used in the applied sciences such as fixed point theory, image and signal processing, medical imaging, decision-making et al. After itroduction of the intuitionistic fuzzy set (IFS), it was used in all areas where FS theory was studied. Park (12) defined IF metric space (IFMS), which is a generalization of FMSs. Park used George and Veeramani's (10) idea of applying t-norm and t-conorm to the FMS meanwhile defining IFMS and studying its basic features. Fixed point theorem for fuzzy contraction mappings is initiated by Heilpern (13). Bose and Sahani (14) extended the Heilpern's study. Alaca et al. (15) are given fixed point theorems related to intuitionistic fuzzy metric spaces(IFMSs). Fixed point results for fuzzy metric spaces and IFMSs are studied by many researchers (16), (17), (18), (19), (20). Kirisci et al. (21, 23) defined neutrosophic contractive mapping and gave a fixed point results in complete neutrosophic metric spaces. In (22), Mohamad studied fixed point aprroach in intuitionistic fuzzy metric spaces. In this paper, we introduce the notion of Neutrosophic Expansion on Neutrosophic Metric Space (NMS) and we prove some fixed point result for Neutrosophic Expansion on NMS.

Preliminaries: Triangular norms (t-norms) (TN) were initiated by Menger (27). In the problem of computing the distance between two elements in space, Menger offered using probability distributions instead of using numbers of distance. TNs are used to generalize with the probability distribution of triangle inequality in metric space conditions. Triangular conforms (t-conorms) (TC) know as dual operations of TNs. TNs and TCs are very significant for fuzzy operations (intersections and unions).

Definition 2.1. Give an operation $\odot$:(0,1)×(0,1)→(0,1). If the operation $\odot$ is satisfying the following conditions, then it is called that the operation $\odot$ is continuous TN (CTN): For s,t,u,∈(0,1),

i) $s\odot 1=s$,
ii) If $s \le u$ and $t \le v$, than $s\odot t\le u\odot v$,
iii)$\odot$ is commutative and associate,
iv)$\odot$ is continuous.

Definition 2.2. Give an operation $\boxdot$:(0,1)×(0,1)→(0,1). If the operation $\boxdot$ is satisfying the following conditions, then it is called that the operation $\boxdot$ is continuous TC (CTC):

i)$\boxdot 0=s$,
ii) If $s\le u$ and $t\le v$, than $s\boxdot t\le u\boxdot v$,
iii)$\boxdot$ is commutative and associate,
iv) $\boxdot$ is continuous.

Remark 2.3.(23) Take $\odot$ and $\boxdot$ are CTN and CTC, respectively. For s,t,v,∈(0,1),

a. If $s>t$, then there are u, such that $s\odot u\ge t$ and $s\ge t\boxdot v$.
b. There are p, such that $t\odot t\ge s$ and $s\ge p\boxdot p$.

Definition 2.4. [28] Take F be an arbitrary set,$\Omega = \{\langle a, HU(a), MU(a), SU(a)\rangle : a\in F\}$ be a NS such that $\Omega: F\times F\times \mathbb{R}+\to[0,1]$. Let $\odot$ and $\boxdot$ show the CTN and CTC, respectively. The four tuple $V=(F, \Omega,\odot,\boxdot)$ is called Multiplicative Neutrosophic Metric Space(MNMS) when the following conditions are satisfied. $\forall a,b,c\in F$,

i) $0 \le H(a, b,\lambda) \le 1$, $0 \le M(a,b,\lambda) \le 1$, $0 \le S(a,b,\lambda) \le 1 \forall\lambda\in\mathbb{R}^+$,
ii) $H(a,b,\lambda)+M(a,b,\lambda)+S(a,b,\lambda) \le 3$, $(for\ \lambda \in \mathbb{R}+)$,
iii) $H(a,b,\lambda) = 1$ $(for\ \lambda > 0)$ $if\ and\ only\ if\ a = b$,
iv) $H(a,b,\lambda) = H(b,a,\lambda)$ $(for\ \lambda > 0)$,
v) $H(a,b,\lambda) \odot H(b,c,\mu) \le H(a,c,\lambda + \mu)$ $(for\ \lambda, \mu > 0)$,
vi) $H(a,b,.):[0,\infty)\to[0,1]$ is continuous,
vii) $\lim\lambda\to\infty\ H(a,b,\lambda) = 1$ $(\forall\ \lambda > 0)$,
viii) $M(a,b,\lambda) = 0$ $(for\ \lambda > 0)$ $if\ and\ only\ if\ a = b$,
ix) $M(a,b,\lambda) = M(b,a,\lambda)$ $(for\ \lambda > 0)$,
x) $M(a,b,\lambda) \boxdot M(b,c,\mu) \ge M(a,c,\lambda + \mu)$ $(for\ \lambda,\mu > 0)$
xi) $M(a,b,.):[0,\infty)\to[0,1]$ is continuous,
xii) $\lim\lambda\to\infty M(a,b,\lambda) = 0$ $(\forall\ \lambda > 0)$,
xiii) $M(a,b,\lambda) = 0$ $(for\ \lambda > 0)$ $if\ and\ only\ if\ a = b$
xiv) $M(a,b,\lambda) = S(b,a,\lambda)$ $(for\ \lambda > 0)$,
xv) $S(a,b,\lambda) \boxdot S(b,c,\mu) \ge S(a,c,\lambda + \mu)$ $(for\ \lambda ,\mu > 0)$,
xvi) $S(a,b,.) :[0,\infty)\to[0,1]$ is continuous,
xvii) $\lim\lambda\to\infty S(a,b,\lambda) = 0$ $(\forall\ \lambda > 0)$
xviii) If $\lambda \le 0$, then $H(a,b,\lambda) = 0$, $M(a,b,\lambda) = 1$, $S(a,b,\lambda) = 1$.

Then $\Omega = (H,M,S)$ is called Multiplicative Neutrosophic Metric (MNM) on F.

Then $\Omega = (H,M,S)$ is called Multiplicative Neutrosophic Metric (MNM) on F.

MAIN RESULTS

Now we define Neutrosophic Expansion on neutrosophic metric space (NMS) and prove fixed point result for it.

3. NEUTROSOPHIC EXPANSION MAPPING:

Definition 3.1. Let V be a MNMS. The mapping f: $F \to F$ is called neutrosophic expansion (NE) if there exists $k \in (1, \infty)$ such that

$$\frac{1}{H(f(a), f(b), \gamma)} - 1 \geq k\left(\frac{1}{H(a, b, \gamma)} - 1\right)$$

$M(f(a), f(b), \lambda) \geq$ k $(M(a, b, \lambda))$,
$S(f(a), f(b), \lambda) \geq$ k $(S(a, b, \lambda))$
for each $a, b \in F$ and $\lambda > 0$.

Theorem: Let V be a complete NMS with (2) in which a NC sequence is a Cauchy sequence. Let f: $F \to F$ is a surjective neutrosophic expansion satisfying conditions of Definition 3.1. Then f has a unique fixed point in V.

Proof: Let $a_0 \in V. Since\ f\ is\ surjective, we\ can\ define\ a\ sequence\ \{a_n\}\ by\ a_n = f(a_{n+1})\ for\ all\ n \in$ ℕ. For each$\gamma > 0$,

$$\frac{1}{H(a_n, a_{n+1}, \gamma)} - 1 = \frac{1}{H(f(a_{n+1}), f(a_{n+2}), \gamma)} - 1$$
$$\geq k\left(\frac{1}{H(a_{n+1}, a_{n+2}, \gamma)} - 1\right)$$

$$\left(\frac{1}{H(a_{n+1}, a_{n+2}, \gamma)} - 1\right) \leq \alpha\left(\frac{1}{H(a_n, a_{n+1}, \gamma)} - 1\right)$$
$$\leq \alpha^n\left(\frac{1}{H(a_1, a_2, \gamma)} - 1\right)$$

In the same way

$M(a_{n+1}, a_{n+2}, \gamma) \leq \alpha^{\,n} M(a_1, a_2, \gamma)$,
$S(a_{n+1}, a_{n+2}, \gamma) \leq \alpha^{\,n}(a_1, a_2, \gamma)$,
where $\alpha = \frac{1}{k} \leq 1$.

Also we have

$$\frac{1}{H(a_n, a_{n+p}, \gamma)} - 1 \leq \frac{1}{*_{i=n}^{p} H\left(a_i, a_{i+1}, \frac{\gamma}{2^{i+1-n}}\right)} - 1$$

$$\leq *_{i=n}^{p}\left(\left(\frac{1}{H\left(a_i, a_{i+1}, \frac{\gamma}{2^{i+1-n}}\right)}\right) - 1\right)$$
$$\leq *_{i=n}^{p} \alpha^{\,i}\left(\frac{1}{H\left(a_0, a_1, \frac{\gamma}{2^{i+1-n}}\right)}\right) - 1$$

Which tends to 0 as $n \to \infty$. So that $\lim_{n\to\infty} H(a_n, a_{n+p}, \gamma) = 1$. In the same way $\lim_{n\to\infty} M(a_n, a_{n+p}, \gamma) = 0$. And $\lim_{n\to\infty} S(a_n, a_{n+p}, \gamma) = 0$. Therefore it is a Cauchy sequence in complete NMS V.Hence $\{a_n\}$ is convergent and converges to some $c \in V$. Now we show that this point c is a neutrosophic fixed point of f. For

.

$$\frac{1}{H(a_{n+1}, f(c), \gamma)} - 1 = \frac{1}{H(f(a_n), f(c), \gamma)} - 1$$

$$\leq \alpha\left(\frac{1}{H(a_n, c, \gamma)} - 1\right) \to 0\ as\ n \to \infty.$$

So that $\frac{1}{H(c, f(c), \gamma)} - 1 = 0\ and\ thus\ H(c, f(c), \gamma) = 1$.

In the same way, we can have

$M(c, f(c), \gamma) = 0 \qquad and \qquad S(c, f(c), \gamma) = 0.$

Therefore $f(c) = c$.

To show the uniqueness, let $f(b) = b$ for some $b \in V$. Then for all $\gamma > 0$, we have

$$\frac{1}{H(c,b,\gamma)} - 1 = \frac{1}{H(f(c),f(b),\gamma)} - 1$$
$$\leq \alpha\left(\frac{1}{H(c,b,\gamma)} - 1\right).$$

Which on repeating yields

$$\frac{1}{H(c,b,\gamma)} - 1 \leq \alpha^n\left(\frac{1}{H(c,b,\gamma)} - 1\right) \to 0 \quad as \quad n \to \infty.$$

Also

$M(c,b,\gamma) = \mathrm{M}(f(c),f(b),\gamma) \leq \alpha M(c,b,\gamma) \leq \alpha^n(M(c,b,\gamma)) \to 0 \quad as \quad n \to \infty\ so\ that\ M(c,b,\gamma) = 0.$
$\mathrm{S}(c,b,\gamma) = \mathrm{S}(f(c),f(b),\gamma) \leq \alpha S(c,b,\gamma) \leq \alpha^n(S(c,b,\gamma)) \to 0 \quad as \quad n \to \infty\ so\ that\ \mathrm{S}(c,b,\gamma) = 0.$
Thus $H(c,b,\gamma) = 1\ and\ M(c,b,\gamma) = S(c,b,\gamma) = 0\ and\ hence\ c = b.$

CONCLUSION

The above theorem is an expansion version to the one Kirisci et al (21) on NMS. Also it opens an era to establish a fixed point theory on NMS.

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