ISSN : 2350-0743

International Journal of Recent Advances in Multidisciplinary Research

Vol. 12, Issue 01, pp. 10646-10656, January, 2025

RESEARCH ARTICLE

ANALYSIS OF COMPETITIVE AND COOPERATIVE RELATIONSHIPS AMONG SECURITIES COMPANIES BASED ON THE 3-SPECIES LOTKA-VOLTERRA MODEL

Changyou Wang* and Yu Zhang

College of Applied Mathematics, Chengdu University of Information Technology, Chengdu, Sichuan, 610225, P. R. China

ARTICLE INFO

Article History

Received 20th October, 2024
Received in revised form
16th November, 2024
Accepted 27th December, 2024
Published online 24th January, 2025

Keywords:

Three-Species; Lotka-Volterra Model; Securities Companies; Competitive and Cooperative Relationships; Grey Estimation Method.

****Corresponding author:***
Changyou Wang

ABSTRACT

There exists an analogical relationship between ecosystems and securities markets. The concept of an ecosystem in the context of capital markets pertains to the intricate relationships and interactions among diverse participants and their surrounding environments. The dynamics of competition and cooperation, transparency, and stability within the securities market are pivotal factors influencing market health and efficiency. This paper undertakes an analysis of the competitive and cooperative dynamics among three securities companies: CITIC Securities, GF Securities, and First Capital Securities, utilizing the three-species Lotka-Volterra model in conjunction with the grey estimation method. The study selects the quarterly operating revenue market share from 2015 to 2023 as the indicator and empirically predicts the evolutionary trends of the market shares of these three companies. The findings reveal that, despite the presence of predatory or competitive relationships in the short term, in the long run, the three companies will coexist in the market, occupying market shares of 13.14%, 6.52%, and 0.67%, respectively. Ultimately, the paper proposes that securities companies should adopt differentiation strategies, deepen supply-side reforms, and augment their service quality. Concurrently, the government should recalibrate policy orientations, strengthen information disclosure obligations, and foster market transparency and stable development.

Citation: Changyou Wang and Yu Zhang. 2025. *"*Analysis of Competitive and Cooperative Relationships Among Securities Companies Based on the 3-Species Lotka-Volterra Model*", International Journal of Recent Advances in Multidisciplinary Research,* 12, (01), 10646-10656.

INTRODUCTION

Ecosystems, as the fundamental units supporting life on Earth, have always been the focus of ecological research due to their complexity and dynamism. Ecological models, which serve as tools for abstracting or formally describing the structure and function of ecosystems, have become crucial for understanding and predicting ecosystem behavior. They not only aid scientists in revealing the internal interactions and energy flows within ecosystems but also play a pivotal role in environmental management and policy formulation. The Lotka-Volterra model, a classic two-species ecological model proposed in 1926, has undergone numerous refinements over the past century in terms of hysteresis, periodicity, persistence, and diversity. In 1980, K. Gopalsamy (1) investigated the impact of time delay on the Lotka-Volterra competition model and demonstrated the conditions for global asymptotic stability and balanced coexistence upon introducing time delay. In 2002, Liu and Chen (2) explored persistence, species extinction, and balanced survival in periodic Lotka-Volterra ecosystem models incorporating time delay, emphasizing hysteresis and periodicity. In 1989, Gilpin and Ayala (3) proposed a nonlinear competitive Lotka-Volterra reaction-diffusion model, building upon the foundational Lotka-Volterra reaction-diffusion model. In 1990, Sun and Cui (4) introduced a novel mathematical model of interspecific competition grounded in nutrient dynamics theory, thereby extending the classic Lotka-Volterra competition equation to provide a broader explanation of interspecific competitive behavior. In 2016, Wang et al. (5) studied a three-species Lotka-Volterra predator-prey model featuring feedback control and ratio-dependence. These refined ecosystem models align more closely with real-world scenarios, offering robust theoretical support for understanding population development patterns and protecting the natural environment. With the widespread application of ecological models across various fields, their theoretical and practical value has been widely recognized. The early Lotka-Volterra model was initially used solely for analyzing species competition in ecosystems, but through research and expansion by scholars, it has now found widespread application in diverse research areas such as sociology and economics, undergoing continuous optimization to better align with

reality. In the field of sociology, in 2018, Marasco and Romano (6) employed the Lotka-Volterra model to study intergenerational conflict in the aging society of the United States from 1974 to 2014, uncovering the impact of different generations on social development. In the realm of the real economy, in 2015, Yang et al. (7) conducted a qualitative and quantitative analysis using grey prediction based on the Lotka-Volterra ecological model to investigate the intrinsic relationships, equilibrium points, and development trends of patent competition among enterprises. Taking the patent competition between Zoomlion Heavy Industry and SANY Heavy Industry as an example, they conducted an empirical analysis and proposed management strategies for patent competition among enterprises. In 2017, Hung et al. (8) predicted the sales of two competitive retail formats using the Lotka-Volterra model, revealing a predator-prey relationship between the two. In 2019, Dominioni et al. (9) explored the competitive and mutually beneficial patterns between oil and renewable energy sources, such as wind and solar power, from 2006 to 2016, discussing the possible reasons behind these patterns and their policy implications. In 2021, Guo et al. (10) analyzed the scale of future energy consumption and pollutant emissions based on the Lotka-Volterra model, predicting changes in the structure of the Chinese passenger car market and its environmental impact. In 2022, Ning et al. (11) constructed a two-agent and three-agent Lotka-Volterra dynamic evolution model for the digital innovation ecosystem to study its symbiotic evolution patterns. Through numerical simulation and verification, they analyzed the symbiotic mode of two-agent core digital enterprise populations and satellite digital enterprise populations, concluding that injecting sustainability into digital innovation ecosystems through university and research institution populations achieves the optimal goal orientation for the evolution of digital innovation ecosystems-a three-agent mutually beneficial symbiotic mode. In 2023, Han et al. (12) proposed an adaptive competition-cooperation model based on an improved Lotka-Volterra model to study the evolution of competition and cooperation between urban rail transit and conventional public transportation, conducting an empirical analysis to verify the feasibility, applicability, and accuracy of the improved model, and providing suggestions for the optimal allocation of urban public transportation. In 2024, Wang et al. (13) constructed a two-agent and three-agent Lotka-Volterra dynamic evolution model for the cold chain logistics system of fruit and nut products, exploring the equilibrium points and stability conditions of two-agent and three-agent systems under different symbiotic modes. Through numerical simulation analysis of the symbiotic evolution trends of two-agent and three-agent systems, they concluded that the three-agent mutually beneficial symbiotic mode represents the optimal state for the symbiotic evolution of the cold chain logistics system of fruit and nut products, embodying true mutually beneficial symbiosis.
The rapid economic development has increasingly underscored the significance of the financial services industry. Numerous scholars have also applied the Lotka-Volterra model within the financial realm. In 2009, Xiong et al. (14) employed the Lotka-Volterra model to analyze trading volume data of Morgan Taiwan Index Futures in Singapore and Taiwan Stock Exchange Index Futures, offering insights into the potential competitive dynamics between CSI 300 Index Futures and FTSE China A50 Index Futures. Their conclusion was that, in the nascent stages of launching financial futures, the two markets are more inclined to be in a mutually beneficial relationship; however, as contract competitiveness and market capacity continue to evolve, so too will their competitive dynamics. In 2012, Comes (15) contemplated utilizing a three-tier Lotka-Volterra model to identify the benchmark equilibrium point for the banking system. In 2019, Khan et al. (16) gathered annual profit data from Indonesian commercial and rural banks and conducted parameter estimation for the Lotka-Volterra model using a genetic algorithm, exploring the competitive landscape between the two from a fractional perspective. In 2020, Mao et al. (17) utilized the Lotka-Volterra model to quantitatively analyze and predict the impact of commercial banks' online payment systems on the development of third-party online payment systems. They concluded that online banking acts as the predator, whereas third-party online payment systems serve as the prey.

Since President Xi Jinping announced the deepening of reforms for the New Third Board and the establishment of the Beijing Stock Exchange in 2021, China's securities market has undergone a series of profound structural transformations. In February 2023, the full implementation of the reform of the stock issuance registration system marked a significant shift in market regulation, transitioning from an approval-based system to a registration-based one. This emphasized the pivotal role of information disclosure and enhanced the degree of marketization. Subsequently, in March 2023, the China Securities Regulatory Commission (CSRC) was reorganized to become a direct subordinate of the State Council, strengthening its capital market regulation responsibilities and incorporating the review duties for corporate bond issuance previously held by the National Development and Reform Commission. This move not only elevated the administrative level and regulatory efficiency of the CSRC but also demonstrated the country's high priority on the high-quality development of the capital market. By 2024, the implementation of the new Company Law provided a fresh legal framework for market operations, encompassing corporate governance, capital operations, and other aspects, marking further refinement of the legal system for China's securities market. This series of reform measures underscores not only the country's recognition of the importance of the securities market but also reflects the evolving trends in market regulation and legal systems. Currently, the application of the Lotka-Volterra model in the securities field primarily focuses on researching the stock market. In 2005, Lee et al. (18) utilized the Lotka-Volterra model to analyze daily empirical index data from the Korea Exchange and the Korea Securities Dealers Automated Quotations, deriving the dynamic relationships between the competitive markets of the two Korean stock exchanges. In 2016, Li et al. (19) employed an extended Lotka-Volterra model to quantitatively study the interactive relationships among multiple tiers of the on-exchange stock market in China, proposing suggestions for optimizing the multi-tiered capital market. In 2017, Zhang et al. (20) used the Lotka-Volterra model to analyze the interactive competition between the Shanghai Stock Market Trading Interconnection Mechanism (Northbound Trading Link) and the Hong Kong Stock Market Trading Interconnection Mechanism (Southbound Trading Link) under the Shanghai-Hong Kong Stock Connect system, finding an unstable predator-prey relationship between the Shanghai and Hong Kong markets. In 2018, He et al. (21) applied the Lotka-Volterra model to study the interaction between the Shanghai-Hong Kong Stock Connect and the Shenzhen-Hong Kong Stock Connect, discovering that after the implementation of the interconnection mechanisms, the Shanghai, Shenzhen, and Hong Kong markets exhibited an overall trend of reciprocal symbiosis and mutual promotion. As securities companies constitute an integral part of the financial services industry, their competitive and cooperative relationships have a profound impact on market stability and industry development. Therefore, it is worthwhile considering applying the Lotka-Volterra model to study the competitive and cooperative relationships among several securities companies. Moreover, to better capture the complex competitive and cooperative relationships among securities companies in the securities market, this paper utilizes a three-species Lotka-Volterra model, combined with actual data, to elucidate the process of competition and cooperation among three securities companies and predict their evolutionary trends.

Lotka-Volterra Model in Securities Market Share Competition

Description and Analysis of the 2-Species Lotka-Volterra Model: Serving as bridges between financing enterprises and investors, securities companies primarily derive their operating revenues from brokerage services, investment banking, asset management, and proprietary trading. Among these, proprietary trading involves securities companies leveraging their own advantages to manage their asset portfolios for profit, which constitutes investment income. Consequently, to better examine the competitive and cooperative relationships among securities companies within the securities market, this paper selects the market share of operating revenue (excluding proprietary trading) of securities companies as an operational indicator, thereby eliminating the interference from proprietary trading.

If only securities companies exist in the securities market, then the growth model of securities companies can be represented by the Logistic equation as follows:

$$\frac{dx}{dt} = rx(1-\frac{r}{s}), \tag{2.1}$$

where x represents the market share of operating revenue for securities company A at time t; r represents the intrinsic growth rate that securities company A can achieve in the current environment; s represents the maximum market share that securities company A can occupy when developing independently in a certain environment. In reality, companies within a particular sector do not exist in isolation, and they interact with each other. Therefore, introducing securities company B, their competition and cooperation model can be described as:

$$\begin{cases} \dfrac{dx_1}{dt} = r_1 \cdot x_1(1-\dfrac{x_1}{s_1}-\dfrac{\alpha_{21}x_2}{s_1}), \\ \dfrac{dx_2}{dt} = r_2 \cdot x_2(1-\dfrac{x_2}{s_2}-\dfrac{\alpha_{12}x_1}{s_2}), \end{cases} \tag{2.2}$$

where x_1, x_2 represent the market shares of operating revenue for securities company A and company B at time t; r_1, r_2 represent the intrinsic growth rates that the two companies can achieve solely based on their own capabilities in the current environment, respectively; s_1, s_2 represent the maximum market shares that the two companies can occupy when developing independently in a certain environment, respectively; α_{21} is the competition coefficients of securities company B towards securities company A; α_{12} is the competition coefficients of securities company A towards securities company B. However, this model can only predict the competition and cooperation relationships between two species. To study the competition and cooperation relationships among multiple species, the three-species Lotka-Volterra model is introduced.

Description and Analysis of 3-Species Lotka-Volterra Model: The Lotka-Volterra competition and cooperation model for 3-species is as follow

$$\begin{cases} \dfrac{dx_1}{dt} = r_1 \cdot x_1(1-\dfrac{x_1}{k_1}-\dfrac{\alpha_{21}x_2}{k_2}-\dfrac{\alpha_{31}x_3}{k_3}), \\ \dfrac{dx_2}{dt} = r_2 \cdot x_2(1-\dfrac{x_2}{k_2}-\dfrac{\alpha_{12}x_1}{k_1}-\dfrac{\alpha_{32}x_3}{k_3}), \\ \dfrac{dx_3}{dt} = r_3 \cdot x_3(1-\dfrac{x_3}{k_3}-\dfrac{\alpha_{13}x_1}{k_1}-\dfrac{\alpha_{23}x_2}{k_2}), \end{cases} \tag{2.3}$$

where x_1, x_2, x_3 represent the market shares of operating revenue for securities company A, company B and company C at time t; r_1, r_2, r_3 represent the intrinsic growth rates that the three companies can achieve solely based on their own capabilities in the current environment, respectively; k_1, k_2, k_3 represent the maximum market shares that the three companies can occupy when developing independently in a certain environment, respectively; $\alpha_{21}, \alpha_{21}/k_2, \alpha_{31}, \alpha_{31}/k_3$ represent the competition coefficients and interaction coefficients, respectively, of securities company B and com- pany C towards securities company A; $\alpha_{12}, \alpha_{12}/k_1, \alpha_{32}, \alpha_{32}/k_3$ represent the competition coefficients and interaction coefficients, respectively, of securities company A and company C towards securities company B; $\alpha_{13}, \alpha_{13}/k_1, \alpha_{23}, \alpha_{23}/k_2$ represent the competition coefficients and interaction coefficients, respectively, of securities company A and company B towards securities company C. Since α_{ij} represents the competition coefficient of the i-th securities company towards the j-th securities company, the competition and cooperation relationships between any two of the three securities companies can be discussed in the following four scenarios When $\alpha_{ij} > 0, \alpha_{ji} > 0$, then a competitive relationship is formed between the i-th securities company and the j-th securities company. The larger the coefficient, the greater the crowding-out effect between the two securities companies; When $\alpha_{ij} < 0, \alpha_{ji} < 0$, then a cooperative relationship is formed between the i-th securities company and the j-th securities company;

- When $\alpha_{ij} > 0, \alpha_{ji} < 0$, then the i-th securities company forms a predator-prey relationship with the j-th securities company, where the development of the j-th securities company promotes the development of the i-th securities company, but the development of the i-th securities company inhibits the development of the j-th securities company;
- When $\alpha_{ij} < 0, \alpha_{ji} > 0$, then the j-th securities company forms a predator-prey relationship with the i-th securities company, where the development of the i-th securities company promotes the development of the j-th securities company, but the development of the j-th securities company inhibits the development of the i-th securities company.

Stability analysis of 3-Species Lotka-Volterra Model: As time passes, ecosystems continuously evolve and ultimately tend towards reaching an equilibrium state. In order to analyze the competition and cooperation relationships among three securities companies, this paper assumes that, as time progresses, the securities market tends towards reaching an equilibrium. At this point, the steady-state equations of the system of differential equations (2.3) can be obtained as follows:

$$\begin{cases} \dfrac{dx_1}{dt} = r_1 \cdot x_1 (1 - \dfrac{x_1}{k_1} - \dfrac{\alpha_{21} x_2}{k_2} - \dfrac{\alpha_{31} x_3}{k_3}) = 0, \\ \dfrac{dx_2}{dt} = r_2 \cdot x_2 (1 - \dfrac{x_2}{k_2} - \dfrac{\alpha_{12} x_1}{k_1} - \dfrac{\alpha_{32} x_3}{k_3}) = 0, \\ \dfrac{dx_3}{dt} = r_3 \cdot x_3 (1 - \dfrac{x_3}{k_3} - \dfrac{\alpha_{13} x_1}{k_1} - \dfrac{\alpha_{23} x_2}{k_2}) = 0. \end{cases} \quad (2.4)$$

By solving the system of equations (2.4), eight equilibrium points can be obtained as follows:

$N_1(0,0,0)$, $N_2(k_1,0,0)$, $N_3(0,k_2,0)$, $N_4(0,0,k_3)$, $N_5(\frac{k_1(1-\alpha_{21})}{1-\alpha_{21}\alpha_{12}}, \frac{k_2(1-\alpha_{12})}{1-\alpha_{21}\alpha_{12}}, 0)$, $N_6(\frac{k_1(1-\alpha_{31})}{1-\alpha_{31}\alpha_{13}}, 0, \frac{k_3(1-\alpha_{13})}{1-\alpha_{31}\alpha_{13}})$, $N_7(0, \frac{k_2(1-\alpha_{32})}{1-\alpha_{32}\alpha_{23}}, \frac{k_3(1-\alpha_{23})}{1-\alpha_{32}\alpha_{23}})$, $N_8(x_1^*, x_2^*, x_3^*)$,

where

$$x_1^* = \frac{k_1(1-\alpha_{21}-\alpha_{31}+\alpha_{31}\alpha_{23}+\alpha_{21}\alpha_{32}-\alpha_{23}\alpha_{32})}{1+\alpha_{12}\alpha_{23}\alpha_{31}+\alpha_{13}\alpha_{21}\alpha_{32}-\alpha_{31}\alpha_{13}-\alpha_{23}\alpha_{32}-\alpha_{21}\alpha_{12}},$$

$$x_2^* = \frac{k_2(1-\alpha_{12}-\alpha_{32}+\alpha_{12}\alpha_{31}+\alpha_{32}\alpha_{13}-\alpha_{31}\alpha_{13})}{1+\alpha_{12}\alpha_{23}\alpha_{31}+\alpha_{13}\alpha_{21}\alpha_{32}-\alpha_{31}\alpha_{13}-\alpha_{23}\alpha_{32}-\alpha_{21}\alpha_{12}},$$

$$x_3^* = \frac{k_3(1-\alpha_{23}-\alpha_{13}+\alpha_{13}\alpha_{21}+\alpha_{23}\alpha_{12}-\alpha_{21}\alpha_{12})}{1+\alpha_{12}\alpha_{23}\alpha_{31}+\alpha_{13}\alpha_{21}\alpha_{32}-\alpha_{31}\alpha_{13}-\alpha_{23}\alpha_{32}-\alpha_{21}\alpha_{12}}.$$

According to the stability theorem of equilibrium points of differential equations, the Jacobian matrix corresponding to the steady-state equations (2.4) can be used to determine the stability of the equilibrium points $N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8$. When all the eigenvalues of the Jacobian matrix have negative real parts, the equilibrium point is stable. The Jacobian matrix corresponding to the steady-state equations (2.4), obtained by computing the partial derivatives, is as follows:

$$D = \begin{bmatrix} \frac{\partial \dot{x}_1(t)}{\partial x_1} & \frac{\partial \dot{x}_1(t)}{\partial x_2} & \frac{\partial \dot{x}_1(t)}{\partial x_3} \\ \frac{\partial \dot{x}_2(t)}{\partial x_1} & \frac{\partial \dot{x}_2(t)}{\partial x_2} & \frac{\partial \dot{x}_2(t)}{\partial x_3} \\ \frac{\partial \dot{x}_3(t)}{\partial x_1} & \frac{\partial \dot{x}_3(t)}{\partial x_2} & \frac{\partial \dot{x}_3(t)}{\partial x_3} \end{bmatrix}. \tag{2.5}$$

Substitute $N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8$ into D to obtain the Jacobian matrix for each equilibrium point. Based on the existence and uniqueness of internal equilibrium points in a 3-dimensional Lotka-Volterra cooperative system proposed by Quan (22) in 1991, and according to Hurwitz's criterion to determine whether all the real parts of the roots are negative.

Assuming $r_1 > 0, r_2 > 0$,and under the premise that all eight equilibrium points are meaningful, the stability conditions for each equilibrium point can be derived based on the values of α_{ij} as follows:

Through the above analysis, it can be concluded that when $r_1 > 0, r_2 > 0$, there are seven scenarios for the evolutionary outcome of the competition and cooperation relationship based on the business income shares of the three securities companies: (1) When $\alpha_{12} > 1$ and $\alpha_{13} > 1$, then N_2 is a stable equilibrium point. As time passes, securities company B and securities company C will withdraw from the securities market; (2) When $\alpha_{21} > 1$ and $\alpha_{23} > 1$, then N_3 is a stable equilibrium point. As time passes, securities company A and securities company C will withdraw from the securities market; (3) When $\alpha_{31} > 1$ and $\alpha_{32} > 1$, then N_4 is a stable equilibrium point. As time passes, securities company A and securities company B will withdraw from the securities market; (4) When $\alpha_{12} > 1$ and $\alpha_{21} > 1$, then N_5 is a stable equilibrium point.As time passes, securities company C will withdraw from the securities market, while securities company A and securities company B will coexist in the securities market with respective sizes of $\frac{k_1(1-\alpha_{21})}{1-\alpha_{21}\alpha_{12}}, \frac{k_2(1-\alpha_{12})}{1-\alpha_{21}\alpha_{12}}$; (5) when $\alpha_{13} > 1$ and $\alpha_{31} > 1$, then N_6 is a stable equilibrium point. As time passes, securities company B will withdraw from the securities market, while securities company A and securities company C will coexist in the securities market with respective sizes of $\frac{k_1(1-\alpha_{31})}{1-\alpha_{31}\alpha_{13}}, \frac{k_3(1-\alpha_{13})}{1-\alpha_{31}\alpha_{13}}$; (6) When $\alpha_{32} > 1$ and $\alpha_{23} > 1$, then N_7 is a stable equilibrium point. As time passes, securities company A will withdraw from the securities market, while securities company B and securit- ies company C will coexist in the securities market with respective sizes of $\frac{k_2(1-\alpha_{32})}{1-\alpha_{32}\alpha_{23}}, \frac{k_3(1-\alpha_{23})}{1-\alpha_{32}\alpha_{23}}$; (7) When $1-\alpha_{21}-\alpha_{31}+\alpha_{31}\alpha_{23}+\alpha_{21}\alpha_{32}-\alpha_{23}\alpha_{32} > 0$, $1-\alpha_{12}-\alpha_{32}+\alpha_{12}\alpha_{31}+\alpha_{32}\alpha_{13}-\alpha_{31}\alpha_{13} > 0$, $1-\alpha_{23}-\alpha_{13}+\alpha_{13}\alpha_{21}+\alpha_{23}\alpha_{12}-\alpha_{21}\alpha_{12} > 0$, $1+\alpha_{12}\alpha_{23}\alpha_{31}+\alpha_{13}\alpha_{21}\alpha_{32}-\alpha_{31}\alpha_{13}-\alpha_{23}\alpha_{32}-\alpha_{21}\alpha_{12} > 0$, then N_8 is a stable equilibrium point. As time passes, the three securities companies will coexist in the securities market.

2. Competitive Analysis of Main Revenue Shares Among Three Securities Firms

(1) Sample Data Selection and Processing

Based on data compiled by China Business Network (23) using Wind information, the 2023 rankings of listed securities companies by operating revenue were obtained. For empirical analysis, this paper selects the top-ranked CITIC Securities Co., Ltd., the fifth-ranked GF Securities Co., Ltd., and the unranked but listed First Capital Securities Co., Ltd. as the three securities companies. The total operating revenue data for the securities market is sourced from the quarterly industry data disclosed by the Securities Association of China (24) from 2015 to 2023. The quarterly operating revenues of the three listed securities companies for the period 2015-2023 are derived from their respective quarterly income statements. Due to missing data for six quarters disclosed by the Securities Association of China, with the missing data being either from the first quarter or the third quarter, the missing parts have been filled using the average of the mid-year and end-of-year data. The specific data is shown in the **Figure 3.1** and **Figure 3.2.**

In order to reflect the competition and cooperation situation of the three securities companies in the securities market, this paper selects the market share of the operating revenues of the three securities companies for each quarter from 2015 to 2023 as the investigation indicator (i.e., the market share of a securities company's operating revenue=the operating revenue of the securities company / the total operating revenue of the securities market). The specific data is shown in the Figure 3.1.

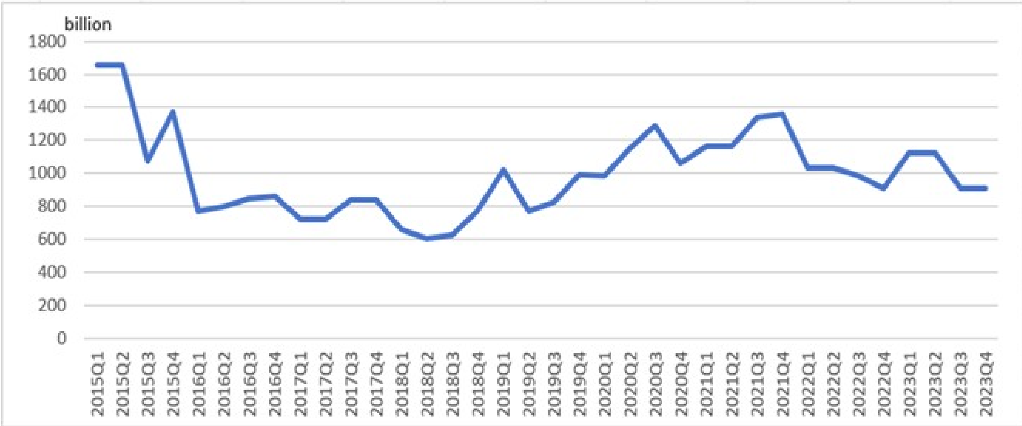

Figure 3.2. Adjusted Total Operating Revenue of Securities Market (2015-2023 Quarters)

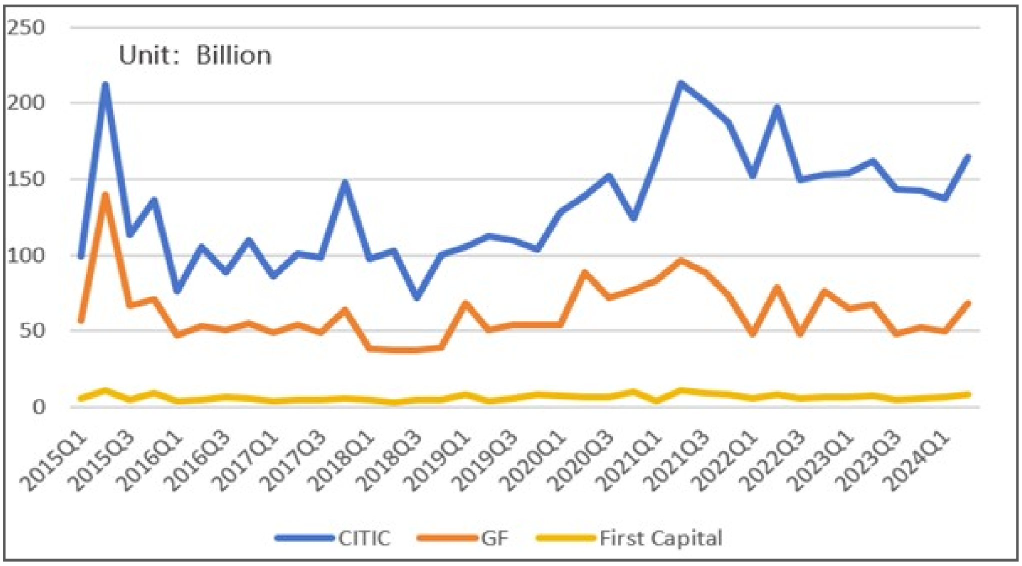

Figure 3.3. Operating revenues of CITIC Securities, GF Securities, and First Capital Securities

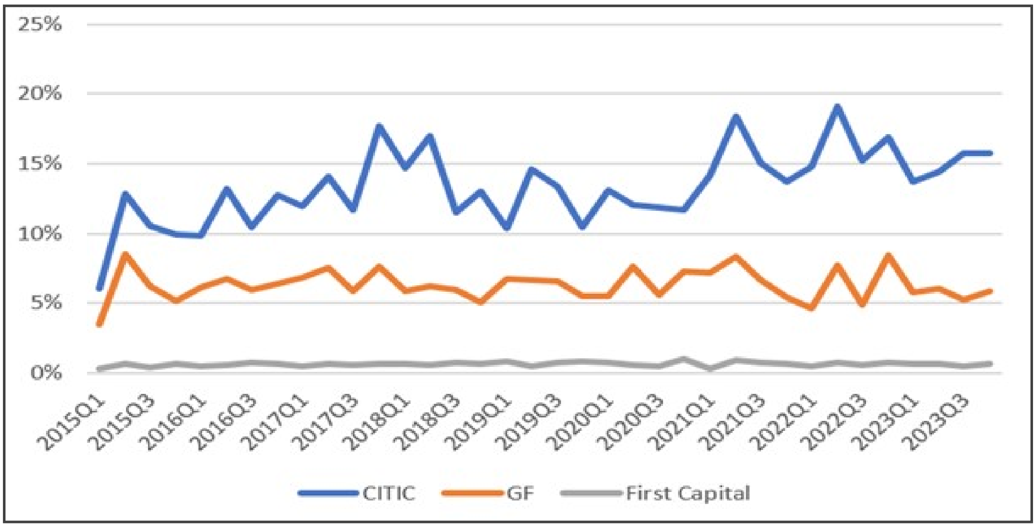

Figure 3.3. Market Share of Operating Revenues: CITIC, GF, & First Capital Securities

Sample Data Selection and Processing: In 2011, Wu and Wang (25) proposed a linear programming method based on the grey direct modeling approach to estimate the parameters of the Lotka-Volterra model, with the aim of minimizing the mean absolute percentage error. The grey estimation method is a prediction technique utilized in dealing with incomplete information systems and is particularly suitable for scenarios where data is scarce. Given the limited data collected in this paper, the grey estimation method is chosen to solve the model. Firstly, the Lotka-Volterra model for three populations, represented by equation (2.3), is converted into the following differential equations

$$\begin{cases} \dfrac{dx_1(t)}{dt} = a_1x_1 + b_1x_1^2 + c_1x_1x_2 + d_1x_1x_3, \\ \dfrac{dx_2(t)}{dt} = a_2x_2 + b_2x_2^2 + c_2x_2x_1 + d_2x_2x_3, \\ \dfrac{dx_3(t)}{dt} = a_3x_3 + b_3x_3^2 + c_3x_3x_1 + d_3x_3x_2. \end{cases} \quad (3.1)$$

From (3.1), it can be obtained that: $r_1 = a_1$, $r_2 = a_2$, $r_3 = a_3$, $k_1 = -a_1/b_1$, $k_2 = -a_2/b_2$, $k_3 = -a_3/b_3$, $\alpha_{21} = (c_1 \times a_2)/(a_1 \times b_2)$, $\alpha_{31} = (d_1 \times a_3)/(a_1 \times b_3)$, $\alpha_{12} = (c_2 \times a_1)/(a_2 \times b_1)$, $\alpha_{32} = (d_2 \times a_3)/(a_2 \times b_3)$, $\alpha_{13} = (c_3 \times a_1)/(a_3 \times b_1)$, $\alpha_{31} = (d_3 \times a_2)/(a_3 \times b_2)$. When the population is very large, it can be concluded that $dx(t)/dt = x(t+1) - x(t)$; using the mapping relationship between the grey derivative and the even logarithm in grey theory, the background value of $dx(t)/dt$ can be derived as $0.5x(t+1) + x(t)$. Therefore, the system of differential equations (3.1) can be transformed into

$$\begin{cases} x_1(t+1) - x_1(t) = a_1\dfrac{x_1(t+1)+x_1(t)}{2} + b_1(\dfrac{x_1(t+1)+x_1(t)}{2})^2 + c_1(\dfrac{x_1(t+1)+x_1(t)}{2})(\dfrac{x_2(t+1)+x_2(t)}{2}) \\ \qquad + d_1(\dfrac{x_1(t+1)+x_1(t)}{2})(\dfrac{x_3(t+1)+x_3(t)}{2}), \\ x_2(t+1) - x_2(t) = a_2\dfrac{x_2(t+1)+x_2(t)}{2} + b_2(\dfrac{x_2(t+1)+x_2(t)}{2})^2 + c_2(\dfrac{x_2(t+1)+x_2(t)}{2})(\dfrac{x_1(t+1)+x_1(t)}{2}) \\ \qquad + d_2(\dfrac{x_2(t+1)+x_2(t)}{2})(\dfrac{x_3(t+1)+x_3(t)}{2}), \\ x_3(t+1) - x_3(t) = a_3\dfrac{x_3(t+1)+x_3(t)}{2} + b_3(\dfrac{x_3(t+1)+x_2(t)}{2})^2 + c_3(\dfrac{x_3(t+1)+x_3(t)}{2})(\dfrac{x_1(t+1)+x_1(t)}{2}) \\ \qquad + d_3(\dfrac{x_3(t+1)+x_3(t)}{2})(\dfrac{x_2(t+1)+x_2(t)}{2}). \end{cases} \quad (3.2)$$

Then, it is converted into a matrix equation system

$$\begin{cases} Y_1 = A_1 \cdot B_1, \\ Y_2 = A_2 \cdot B_2, \\ Y_3 = A_3 \cdot B_3. \end{cases} \quad (3.3)$$

where

$$Y_1 = \begin{bmatrix} x_1(2)-x_1(1) \\ x_1(3)-x_1(2) \\ \vdots \\ x_1(36)-x_1(35) \end{bmatrix}, Y_2 = \begin{bmatrix} x_2(2)-x_2(1) \\ x_2(3)-x_2(2) \\ \vdots \\ x_2(36)-x_2(35) \end{bmatrix}, Y_2 = \begin{bmatrix} x_3(2)-x_3(1) \\ x_3(3)-x_3(2) \\ \vdots \\ x_3(36)-x_3(35) \end{bmatrix}, B_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix}, B_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix}, B_3 = \begin{bmatrix} a_3 \\ b_3 \\ c_3 \\ d_3 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \dfrac{x_1(2)+x_1(1)}{2} & (\dfrac{x_1(2)+x_1(1)}{2})^2 & (\dfrac{x_1(2)+x_1(1)}{2})(\dfrac{x_2(2)+x_2(1)}{2}) & (\dfrac{x_1(2)+x_1(1)}{2})(\dfrac{x_3(2)+x_3(1)}{2}) \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{x_1(36)+x_1(35)}{2} & (\dfrac{x_1(36)+x_1(35)}{2})^2 & (\dfrac{x_1(36)+x_1(35)}{2})(\dfrac{x_2(36)+x_2(35)}{2}) & (\dfrac{x_1(36)+x_1(35)}{2})(\dfrac{x_3(36)+x_3(35)}{2}) \end{bmatrix},$$

$$A_2 = \begin{bmatrix} (\frac{x_2(2)+x_2(1)}{2}) & (\frac{x_2(2)+x_2(1)}{2})^2 & (\frac{x_2(2)+x_2(1)}{2})(\frac{x_1(2)+x_1(1)}{2}) & (\frac{x_2(2)+x_2(1)}{2})(\frac{x_3(2)+x_3(1)}{2}) \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_2(36)+x_2(35)}{2} & (\frac{x_2(36)+x_2(35)}{2})^2 & (\frac{x_2(36)+x_2(35)}{2})(\frac{x_1(36)+x_1(35)}{2}) & (\frac{x_2(36)+x_2(35)}{2})(\frac{x_3(36)+x_3(35)}{2}) \end{bmatrix},$$

$$A_3 = \begin{bmatrix} \frac{x_3(2)+x_3(1)}{2} & (\frac{x_3(2)+x_3(1)}{2})^2 & (\frac{x_3(2)+x_3(1)}{2})(\frac{x_1(2)+x_1(1)}{2}) & (\frac{x_3(2)+x_3(1)}{2})(\frac{x_2(2)+x_2(1)}{2}) \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_3(36)+x_3(35)}{2} & (\frac{x_3(36)+x_3(35)}{2})^2 & (\frac{x_3(36)+x_3(35)}{2})(\frac{x_1(36)+x_1(35)}{2}) & (\frac{x_3(36)+x_3(35)}{2})(\frac{x_2(36)+x_2(35)}{2}) \end{bmatrix}.$$

Finally, based on the least squares criterion, the parameter estimates can be obtained as follows

$$\begin{cases} B_1 = (A_1^T A_1)^{-1} A_1^T Y_1, \\ B_2 = (A_2^T A_2)^{-1} A_2^T Y_2, \\ B_3 = (A_3^T A_3)^{-1} A_3^T Y_3. \end{cases} \quad (3.4)$$

*S*ubstitute the market shares of the operating revenues of the three securities companies for each quarter from 2015 to 2023 into Equation (3.2). Then, use MATLAB to solve for the parameters based on Equation (3.4). The results are shown in Table 3.1.

Table 3.1 Parameter estimation for the three-species Lotka-Volterra model

Parameter	Estimated value
r_1	0.6556
r_2	0.6922
r_3	0.3569
k_1	1.8941
k_2	0.1702
k_3	0.0096
α_{21}	-0.3859
α_{31}	1.5430
α_{12}	3.1214
α_{32}	0.5724
α_{13}	-2.9185
α_{23}	1.3142

From this, the estimated results for the three-species Lotka-Volterra model (2.3) can be obtained as Using MATLAB, plot the fitted data for the Lotka-Volterra model of three populations and estimate the market share of operating revenues for three securities companies from 2015 to 2069 as shown in Figure 3.4 and Table3.1.

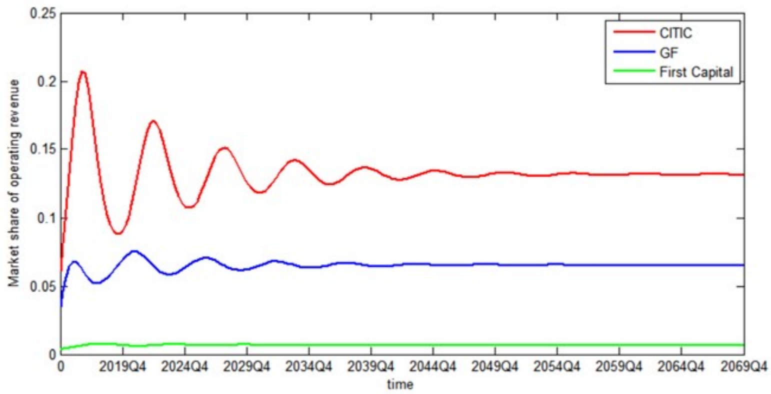

Figure 3.5. The fitted data plot for the three-species Lotka-Volterra model

Table 3.2. Quarterly Data Fitted by Three-Species Lotka-Volterra Model

Time	CITIC	GF	First Capital
2068Q1	0.1317	0.0652	0.0067
2068Q2	0.1316	0.0652	0.0067
2068Q3	0.1315	0.0652	0.0067
2068Q4	0.1315	0.0652	0.0067
2069Q1	0.1314	0.0652	0.0067
2069Q2	0.1314	0.0652	0.0067
2069Q3	0.1313	0.0652	0.0067
2069Q4	0.1313	0.0652	0.0067

Analysis of empirical conclusions: From the perspective of parameter estimation results, $r_1 = 0.6556 > 0$, $r_2 = 0.6922 > 0$, $r_3 = 0.3569 > 0$ satisfy the assumptions of stability analysis. Since $1-\alpha_{21}-\alpha_{31}+\alpha_{31}\alpha_{23}+\alpha_{21}\alpha_{32}-\alpha_{23}\alpha_{32} = 0.8975 > 0$ and $1-\alpha_{12}-\alpha_{32}+\alpha_{21}\alpha_{31}+\alpha_{32}\alpha_{13}-\alpha_{31}\alpha_{13} = 4.9552 > 0$ and $1-\alpha_{23}-\alpha_{13}+\alpha_{13}\alpha_{21}+\alpha_{23}\alpha_{12}-\alpha_{21}\alpha_{12} = 9.0371 > 0$ and $1+\alpha_{12}\alpha_{23}\alpha_{31}+\alpha_{13}\alpha_{21}\alpha_{32}-\alpha_{31}\alpha_{13}-\alpha_{23}\alpha_{32}-\alpha_{21}\alpha_{12} = 12.9298 > 0$. Thus, the equilibrium points N_8 determined to be stable. As time passes, the market shares of the operating revenues of the three securities companies will all increase and eventually tend towards the equilibrium point $N_8(0.1314, 0.0652, \ , 0.0067)$. The three securities companies will coexist in the securities market with operating revenue shares of 13.14%, 6.52%, and 0.67% respectively. Parameters $\alpha_{21} = -0.3859 < 0, \alpha_{12} = 3.1214 > 0$ indicates that CITIC Securities will prey on GF Securities in the short term, and the development of CITIC Securities will inhibit the development of GF Securities, but the development of GF Securities will promote the development of CITIC Securities. Parameters $\alpha_{31} = 1.5430 > 0, \alpha_{13} = -2.9185 < 0$ indicates that First Capital Securities will prey on CITIC Securities in the short term. Parameters $\alpha_{32} = 0.5724 > 0, \alpha_{23} = 1.3142 > 0$ indicates that GF Securities and First Capital Securities will be in a competitive relationship in the short term, and GF Securities will have a greater crowding-out effect on First Capital Securities.

CONCLUSIONS AND SUGGESTIONS

This paper applies the three-species Lotka-Volterra model of interspecific relationships within ecosystems, utilizing the market share of operating revenues of securities companies as an indicator, to analyze the competitive dynamics among CITIC Securities Co., Ltd., GF Securities Co., Ltd., and First Capital Securities Co., Ltd. The results indicate the following: (1) Based on the analysis of existing data, there exist predation or competition relationships between any two of the three securities companies. However, applying the three-species Lotka-Volterra model leads to the conclusion that these three securities companies will ultimately coexist in the securities market. Therefore, the three-species Lotka-Volterra model offers a more realistic depiction than the two-species Lotka-Volterra model and provides a clearer understanding of the dynamic evolution of the market share distribution among these three securities companies within the total operating revenue of the securities market. (2) Based on the analysis of existing data, over a specific period of time, the market shares of the operating revenues of the three securities companies have all increased, indicating a certain degree of mutual restraint among them. However, as time progresses, these three securities companies will converge towards an equilibrium point $N_8(0.1314, 0.0652, 0.0067)$, where they will occupy 13.14%, 6.52%, and 0.67% of the operating revenue shares in the securities market, respectively. With the implementation of the registration-based IPO system, securities companies bear greater responsibilities in the listing process, and healthy competition among them becomes particularly important. Healthy competition can enhance the quality of listed companies, improve the service quality of the securities market, provide adequate funding for genuinely development-needy enterprises, offer more efficient returns to investors, and stimulate the vitality of the securities market. Based on the analysis results of the competitive relationships among the securities companies, this paper proposes the following suggestions: (1) The market shares of operating revenues for CITIC Securities, GF Securities, and First Capital Securities will tend towards balance, indicating that their competitiveness in brokerage, investment advisory, and consulting businesses within the securities market is relatively equal. With the continuous opening up of the securities market, securities companies of varying sizes should seek their own unique development paths in the intense market competition to achieve coexistence. Therefore, small and medium-sized securities companies should implement differentiation strategies based on their unique development characteristics, deepen supply-side reforms, and provide superior financial services to investors and financing enterprises. This could involve reducing the proportion of traditional businesses and leveraging financial technology to offer personalized investment and asset management advice and services tailored to their specific strengths. Leading securities companies can rapidly expand their business scale and market competitiveness through mergers and acquisitions, restructuring, and other strategic methods. Securities companies should also aim to increase their proprietary trading revenues by delivering high-quality services and bolster their capital base to strengthen their risk management capabilities. (2) In the context of the registration-based IPO system, securities companies play a pivotal role in the corporate listing process, particularly in the quality control of information disclosure. Given the increasingly significant responsibility of securities companies in advising investors on the quality of listed companies and in ensuring accurate information disclosure, it is recommended that the government adjust its policy orientation to focus more intently on securities companies' performance in the information disclosure process when evaluating their overall performance. Such a policy shift will incentivize securities companies to more diligently fulfill their information disclosure obligations and refrain from engaging in unhealthy competition, thereby enhancing market transparency, protecting investors, and fostering the stable development of the securities market. Regulatory authorities also need to strengthen their oversight of market structure to prevent monopolistic behaviors by securities companies in the securities market. Additionally, they should provide business training and policy incentives to small securities companies to ensure balanced development across the securities industry.

Availability of data and materials: The data used in this article comes from publicly available data on Chinese government websites.

Competing interests: The authors declare no competing interests.

Authors' contributions: C. Y. Wang and Y. Zhang contributed equally to each part of this article.

REFERENCES

Gopalsamy, K. Time lags and global stability in two-species competition, Bulletin of Mathematical Biology, 42(5) (1980) 729-737.

Liu, S. Q. L. S. Chen, Permanence, extinction and balancing survival in nonautonomous Lotka-Volterra system with delays, Applied Mathematics and Computation, 129(2-3) (2002) 481-499.

Gilpin, M. E. F. J. Ayala, Global Models of Growth and Competition, Proceedings of the National Academy of Sciences, 70(12) (1973) 3590-3593.

Sun, G. W. Q. W. Cui, A new mathematical model of interspecific competition: an expansion of Lotka-Volterra competition equations, Journal of Applied Ecology, 1(1) (1990) 31-39. (In Chinese).

Wang, C. Y. H. Liu, S. Pan, et al., Globally Attractive of a Ratio-Dependent Lotka-Volterra Predator-Prey Model with Feedback Control, Advances in Bioscience and Bioengineering, 4(5) (2016) 59-66.

Marasco, A. A. Romano, Deterministic modeling in scenario forecasting: estimating the effects of two public policies on intergenerational conflict, Quality & Quantity, 52(5) (2018) 2345-2371.

Yang, P. M. Wang, L. L. Zhu. Enterprise Patent Competition Analysis Based on Lotka-Volterra Model, Journal of Intelligence, 34(12) (2015) 85-90. (In Chinese).

Hung, H. C. Y. C. Chiu, H. C. Huang, M. C. Wu, An enhanced application of Lotka-Volterra model to forecast the sales of two competing retail formats, Computers & Industrial Engineering, 109 (2017) 325-334.

Dominioni, G. A. Romano, C. Sotis, A Quantitative Study of the Interactions between Oil Price and Renewable Energy Sources Stock Prices, Energies, 12(9) (2019) 1693-1704.

Guo, D. W. Yan, X. B. Gao, et al, Forecast of passenger car market structure and environmental impact analysis in China, Science of the Total Environment, 772 (2021), Article ID: 14490.

Ning, L. J. J. T. Liu, Y. X. Xiao, D. J. Kong, Research on symbiosis model of digitalinnovation ecosystem, Studies in Science of Science, 40(08) (2022) 1481-1494. (In Chinese).

Han, B. R. H. J. Pu, Z. J. Zhu, Evolution of the Competition-Cooperation Relationship between Urban Rail Transit and Conventional Bus Based on the Improved Lotka-Volterra Model, Journal of Chongqing Jiaotong University(Natural Science), 42(02) (2023) 106-112. (In Chinese).

Wang, Z. W. X.Y. Deng, Y. Pang, Three-subject symbiotic evolution relationship of forest and fruit products cold chain logistics system, Journal of Central South University of Forestry & Technology, 44(05) (2024) 191-202. (In Chinese).

Xiong, X. W. Zhang, S. Li, et al, Competitive analysis of index futures markets based on Lotka-Volterra model, Journal of Systems Engineering, 24(05) (2009) 581-588. (In Chinese).

Comes, C. A. Banking system: three level Lotka-Volterra model, Procedia Economics and Finance, 3 (2012) 251-255.

Khan, M. A. M. Azizah, S. Ullah, A fractional model for the dynamics of competition between commercial and rural banks in Indonesia, Chaos, Solitons & Fractals, 122 (2019) 32-46.

Mao, S. H. M. Zhu, X. P. Wang, X. P. Xiao, Grey-Lotka-Volterra model for the competition and cooperation between third-party online payment systems and online banking in China, Applied Soft Computing, 95 (2020), Article ID: 106501.

Lee, S. J. D. J. Lee, H. S. Oh, Technological forecasting at the Korean stock market: A dynamic competition analysis using Lotka-Volterra model, Technological Forecasting and Social Change, 72(8) (2005) 1044-1057.

Peng, J. Y. Li, W. H. T. H. Liu, The Interactive Board-Level Relationship of Stock Exchange Market in China: Based on the Perspective of Interspecific Relationship, Journal of Financial Research, 05 (2016) 82-96. (In Chinese).

Zhang, B. Z. G. Yao, P. F. Song, Research on the Dynamic Interaction Relationship in the Shanghai-Hong Kong Stock Market Communication: Based on the Lotka-Volterra Financial Ecology Model, South China Finance, 04 (2017) 49-57. (In Chinese).

He, S. X. Y. Lin, X. Y. Zhang, et al, Research on Interaction between Shanghai-HongKong Stock Connect and Shenzhen-Hong Kong Stock Connect on the Bckground of Interconnection Mechanism, Exploration of Financial Theory, 06 (2018) 43-53. (In Chinese).

Quan, H. S. The Global Stability of Three-Dimensional Lotka-Volterra Co-operation System Mathematica Applicata, 01 (1991) 53-57. (In Chinese).
YICAI, Overview of Performance of 23 Listed Securities Firms: Half of the Top Firms Experience Declines in Both Revenue and Net Profit, While Small and Medium-sized Securities Firms Deliver Impressive Performances(DB/OL), https://www.yicai.com/news/102056175.html. (In Chinese).
Securities Association of China, China Securities Depository and Clearing Corporation Limited releases operating data of securities companies(DB/OL), https://www.sac.net.cn/hysj/jysj/. (In Chinese).
Wu, L. F. Y. N. Wang, Estimation the parameters of Lotka-Volterra model based on grey direct modelling method and its application, Expert Systems with Applications, 38(6) (2011) 6412-6416.

****