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RESEARCH ARTICLE

CONSTRUCTION OF A PROBABILTY FUNCTION RELATED TO CODES GENERATED BY SYMMETRIC DESIGN

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ABSTRACT

For an odd integer k and C code of (v, k, λ)-symmetric design, in subset C_1, by defining X code word as a relative weight, maximum relative weight and the ratio of relative weight, new results were proved. As a result of this, relative probability p was found.

INTRODUCTION

Symmetric design is a concept in algebraic combinatorics and design theory. A symmetric design is a mathematical structure that involves a set of points and a collection of subsets of those points, known as blocks. These blocks are arranged in a way that satisfies certain symmetry and intersection properties.

Formally, a (v, k, λ)-symmetric design is defined by three parameters:

v: The number of points in the design.
k: The size of each block (subset of points).
λ: The number of blocks containing any given pair of points.

The design is called symmetric because it possesses certain symmetrical properties. Specifically, for any pair of points, there is a fixed number of blocks containing that pair, and this number is the same for all pairs of points. This helps ensure a balance and regularity within the design. Symmetric designs find applications in various fields, including coding theory, statistics, and experimental design. They are used to efficiently organize experiments, surveys, and other data collection methods, ensuring a balanced and systematic approach to data analysis while minimizing bias. A symmetric design in algebraic combinatorics is a planned arrangement of points and blocks that abides by particular symmetry and intersection qualities, and it has applications in many branches of mathematics as well as outside of them [1, 2]. Many researchers dealing with symmetrical design argue that the symmetrical design is applicable to the organization of experiments in statistics.The creation of a (v, k, λ)-parameter design class is important as a field of use in statistics. After giving some important definitions and properties of symmetrical design, in the last section a probability function related to (v, k, λ)-parameter symmetric design is found.

(v,k,λ)-parameter Symmetric Design Definition and Symmetrical Design Code:

The (v, k, λ)-parameter symmetric block design is an overlap structure that satisfies the following axioms [3].

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- There are v as many points.
- There are v as many blocks.
- Each point coincides with k blocks.
- Each block coincides with k points..
- The number of points where any two blocks coincide is λ.
- The number of blocks in which any two points coincide is λ .
- n=k-λ is called the order of the integer (v, k, λ)-parameter symmetric design[3].

Let $P=\{p_1, p_2, \ldots, p_v\}$ be the set of points and $B=\{B_1, B_2, \ldots, B_v\}$ the set of blocks. In a (v, k, λ)-parameter symmetric design, the matrix $A=[a_{ij}]_{v\times v}$ whose elements are

$$a_{ij}=\begin{cases}1 & \text{If the point } p_j \text{coincides with the } B_j \text{ block.}\\ 0 & Otherwise\end{cases}$$

is called the incidence matrix of the symmetric design [4]. The code of(v, k, λ)-parameter symmetric design is the subspace of the design produced by the rows of the A incidence matrix [5].

Relative Weight: In this section, for the odd number k and C code of (v,k,λ)-parameter symmetric design, the relative weight, the maximum relative weight, and the relative weight of the word X in the C_1 subset ratio are defined and the relative probability of p is found.

Definition: For an odd integer k, let C_1 subset of C code of (v, k, λ)-symmetric design be

- Rows of incidence matrix of the symmetric design,
- Row vectors obtained by taking 1 instead of 0s and 0 instead of 1s in incidence matrix,
- Set of words 0=(0 0 … 0) and 1=(1 1 … 1).

Let consider the a vector X of C_1 . If the positions of the 1s in this word are $s_1,s_2,...,s_k$ let's call the sum of $s_1+s_2+...s_k$ the relative weight of the code word X and Let's denote this with $S(X)=s_1+s_2+...s_k$. In this case, when $1=(11...1)\in C_1$, its relative weight is $1+2+3+...+v=\frac{v(v+1)}{2}$. This can be called the maximum relative weight of the C code of symmetric design defined in this way. Let's use the notation $e(X) = \frac{S(X)}{\text{maximum relative weight}}$ for a code word $X\in C_1$.We can call this the relative weight ratio of the word. It can be easily seen that if $X_i, X_j\in C_1$ is $e(X_i +X_j)=e(X_i)+e(X_j)-2e(X_i . X_j)$.

Theorem: If the code of the (v,k,λ)-symmetric design, defined as above, is C, and the code words $X_1,X_2,...,X_v$, which consists of the rows of the incidence matrix,

$\sum_{i=1}^{v} e(X_i)=k$.

Proof:

$$\sum_{i=1}^{v} e(X_i) = e(X_1) + e(X_2) + \cdots + e(X_v)$$

$$=\frac{S(X_1)}{\text{max. rel. w.}}+\frac{S(X_2)}{\text{max. rel. w.}}+\ldots+\frac{S(X_v)}{\text{max. rel. w.}}$$

$$=\frac{S(X_1)+S(X_2)+\cdots S(X_v)}{\text{maximum relative weight}}$$

$$=\frac{S(X_1)+S(X_2)+\cdots S(X_v)}{\frac{v(v+1)}{2}}$$

From the definition of the (v,k,λ)-symmetric design, there are exactly k 1's in each position in the v code word, which consists of the lines of the coincidence matrix. That is, since there are k "1s" in the first position, k "1"s, in the second position,…,k "1s" in the vnth position, for the code word v

$$S(X_1) + S(X_2) + \cdots S(X_v) = k(1 + 2 + \cdots + v)$$

$= k\frac{v(v+1)}{2}$

is found. Finally $\sum_{i=1}^{v} \mathrm{e}(X_i)$=k.

Theorem: Where k is an odd number, the row vectors of the incidence matrix of the complement of the (v,k,λ)-symmetric design are also the code words of the binary C code of the design. Proof: Since there will be k "1s" in each column of the A incidence matrix of the (v,k,λ)-symmetric design,

(k,k,…,k)≡(1,1,…,1) (mod2)

is obtained by summing the v rows of A (k=2t+1≡1(mod2)).Then the vector X+(1,1,...,1) with a row vector X of A is a code word of C and belongs to the set of row vectors of the design's complement.

Theorem: Where k is an odd number, if the D' is the complement of the (v,k,λ)-symmetric design and the code words consisting of the rows of its the incidence matrix are $X'_1, X'_2, \ldots, X'_v$ then

$\sum_{i=1}^{v} \mathrm{e}(X'_i) = v - k.$

Proof:

$$\sum_{i=1}^{v} \mathrm{e}(X_i) = \mathrm{e}(X_1) + \mathrm{e}(X_2) + \cdots + \mathrm{e}(X_v)$$

$$= \frac{S(X_1')}{\text{max. rel. w.}} + \frac{S(X_2')}{\text{max. rel. w.}} + \ldots + \frac{S(X_v')}{\text{max. rel. w.}}$$

$$= (v-k)\frac{\frac{v(v+1)}{2}}{\frac{v(v+1)}{2}} = v - k.$$

Where k is an odd number, since the set of code words C_1 consists of the row vectors of the coincidence matrix of the (v,k,λ)-symmetric design, the row vectors of the coincidence matrix of the design's complement, and the vectors (0,0,...,0) , (1,1,...,1) $\sum_{X_i \in C_1} e(X_i) = v + 1$.

Since $\frac{1+2+3+\cdots+v}{1+2+3+\cdots+v} = 1$ for X_1=(1 1 1 …1),$\mathrm{e}(X_1) = 1$.

$\sum_{X_i \in A} e(X_i) = k, \ \ \sum_{X_i' \in A'} e(X_i') = v - k,$

$\sum_{X_i \in C_1} e(X_i) = k + v - k + 1 = v + 1$ is obtained.

Definition: Let's denote a set of relative probabilities with C_0, which we will define as follows for C_1, the set obtained by adding 0 and 1 to the set of rows of the incidence matrix A of the design D and the rows of the complement of D' to the incidence matrix A'.Let the relative probability of the code word X_i be defined as

$$p(X_i) = \frac{S(X_i)}{(\mathrm{v}+1)(\text{maximum relative weight})}.$$

Since the maximum relative weight is $\frac{v(v+1)}{2}$,

$$p(X_i) = \frac{S(X_i)}{(\mathrm{v}+1)\frac{v(v+1)}{2}},$$

$$p(X_i) = \frac{2S(X_i)}{v(v+1)^2}.$$

Theorem:

$\sum_{X_i \in C_1} p(X_i) = 1.$

Proof:

Sum of p's for code words corresponding to blocks of symmetrical design

$$\frac{2k\frac{v(v+1)}{2}}{v(v+1)^2} = \frac{k}{v+1} \quad \text{(for row vectors of A)}, \qquad (1)$$

The sum of p's for the code words obtained by writing 1s instead of 0s and 0s instead of 1s in each block in the symmetrical design

$$\frac{2(v-k)\frac{v(v+1)}{2}}{v(v+1)^2} = \frac{v-k}{v+1} \text{ (for row vectors of A')}, \qquad (2)$$

for code word (111…1)

$$\frac{2\frac{v(v+1)}{2}}{v(v+1)^2} = \frac{1}{v+1}\,. \qquad (3)$$

Adding (1), (2), and (3) gives

$$\sum_{X_i \in C_1} p(X_i) = \frac{k}{v+1} + \frac{v-k}{v+1} + \frac{1}{v+1} = 1\,.$$

CONCLUSION

The designs used in arranging experiments in statistics developed as a result of mathematicians' use of elements of algebra. Designs have become the field of study of mathematicians. This study also made a contribution to this field. The creation of a (v,k,λ)-parameter design class is very important as a field of use in statistics. The code for the (v,k,λ)-parameter symmetric design is the subspace of the design produced by the rows of the overlap matrix A. For the C code of the (v,k,λ)-parameter symmetric design where k is an odd number, the relative weight, maximum relative weight and relative weight ratio of the word X in the C_1 subset are defined and its properties are examined. Using these, the relative probability p was found.

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